Test Corrections on Test #2

Due Nov 18

Test 1 Corr. due today!!

1. Redo any question on separate paper.
2. Explain math mistakes made on test.

→ If both are good → get $\frac{1}{2}$ pts back.

3. Can redo as many times as you want.
4. You must work alone or with me only.

Question: A coin is flipped 10 times. How many different outcomes result in less than 4 tails? What is the chance of that happening?

Total # of outcomes (no restrictions)

$= 2 \cdot 2 \cdots 2 = 2^{10}$ = denominator in probability

$= 1024$

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3 of these tails

$\binom{10}{3}$ = number of ways to chose 3 of the 10 ten times to get Tails.

# of way to get < 4 Tails:

$$\binom{10}{3} + \binom{10}{2} + \binom{10}{1} + \binom{10}{0}$$

$$120 + 45 + 10 + 1 = 176$$

$$\frac{10 \cdot 9}{2 \cdot 1} = 45 \qquad \frac{\cancel{10}^{5} \cdot \cancel{9}^{3} \cdot 8}{\cancel{3} \cdot \cancel{2} \cdot 1} = 120$$

$$\text{Chance of getting} < 4 \text{ Tails} = \frac{176}{1024} = \frac{88}{512} = \boxed{\frac{11}{64}}$$

Things We notice from Pascal's Triangle:

& Binomial Theorem

$$(A+B)^n = \sum_{k=0}^{n} \binom{n}{k} A^k B^{n-k}$$

1

1 2 1

1 3 3 1

1 4 6 4 1

1 5 10 10 5 1

sum $32 = 2^5$

Let $A=1, B=1$

$$\boxed{2^n = \sum_{k=0}^{n} \binom{n}{k}}$$

$$2^5 = \binom{5}{0} + \binom{5}{1} + \binom{5}{2} + \binom{5}{3} + \binom{5}{4} + \binom{5}{5}$$

Symmetry

$$\binom{5}{1} = \binom{5}{4}$$

$$\binom{5}{2} = \binom{5}{3}$$

$$\boxed{\binom{n}{k} = \binom{n}{n-k}}$$

Pascal Identity.

$$\boxed{\binom{n+1}{k} = \binom{n}{k} + \binom{n}{k-1}}$$

Can we prove it?

Proof: Method 1

$$LHS = \binom{n+1}{k} = \frac{(n+1)!}{k!\,\underbrace{(n+1-k)!}_{\text{cancel}}} = \frac{(n+1)(n)(n-1)\cdots(n-k+2)}{k(k-1)(k-2)\cdots 1}$$

$$\text{RHS} = \frac{n!}{k!\,(n-k)!} + \frac{n!}{(k-1)!\,(n-k+1)!}$$

$$= \frac{n(n-1)(n-2)\cdots(n-k+1)}{k(k-1)(k-2)\cdots 1} + \frac{k\,n(n-1)(n-2)\cdots(n-k+2)}{k\,(k-1)(k-2)\cdots 1}$$

$$= \frac{n(n-1)(n-2)\cdots \overset{(n-k+2)}{\times}(n-k+1)}{k!} + \frac{n(n-1)(n-2)\cdots(n-k+2)k}{k!}$$

$$= \frac{\underline{n(n-1)(n-2)\cdots(n-k+2)}(n-k+1) + \underline{n(n-1)(n-2)\cdots(n-k+2)}k}{k!}$$

$$= \frac{n(n-1)(n-2)\cdots(n-k+2)\left[\overbrace{(n-k+1) + k}^{n+1}\right]}{k!}$$

$$= \frac{(n+1)n(n-1)\cdots(n-k+2)}{k!}$$

$$= \binom{n+1}{k} = \text{LHS}. \quad \blacksquare$$

Is there another way to prove $\binom{n+1}{k} = \binom{n}{k} + \binom{n}{k-1}$?

<u>Yes</u> Induction on k: $k=0$ $\binom{n+1}{0} = 1$,

$\binom{n}{0} = 1$ $\binom{n}{-1} = 0$.

Since $1 = 1+0$, the eq. is true for $k=0$.

(also $k=1$ is easy: $\binom{n+1}{1} = n+1$, $\binom{n}{1} + \binom{n}{0} = n+1$. ✓)

Induction hypothesis: Suppose we know formula for $k = s$ (and lower)

$$\binom{n+1}{s} = \binom{n}{s} + \binom{n}{s-1}$$

Then consider.

$$\binom{n+1}{s+1} = \frac{(n+1)!}{(s+1)!(n-s)!} = \frac{(n+1)\, n!}{(s+1)\, s!\,(n-s)!}$$

$$\cancel{= \frac{n+1}{s+1}\binom{n}{s}} \quad \frac{(n+1-s)}{s+1}\,\frac{(n+1)!}{s!\,(n+1-s)(n-s)!}$$

$$\cancel{= \frac{n+1}{s+1}} \quad = \frac{(n+1-s)}{s+1}\binom{n+1}{s}$$

By the induction hypothesis

$$= \frac{n+1-s}{s+1}\left[\binom{n}{s} + \binom{n}{s-1}\right]$$

$$= \frac{n+1-s}{s+1}\left[\frac{n!}{s!\,(n-s)!} + \frac{n!}{(s-1)!\,(n-s+1)!}\right]$$

...

After messing around, this might work. ☺

Method 3

Quiz

1. Favorite Restaurant in Fort Worth.
2. $\binom{n}{k}$ is the number of ways to

____.

3. Give the formula for $\binom{n}{k}$.
4. State the Pigeonhole Principle.